MAT 4575 M. ALVO

Solution au Devoir 1

sur 12

1. 6.1.1 $f(x;\theta) = \frac{1}{\Gamma(\alpha)\,\beta^{\alpha}} \cdot x^{\alpha-1} e^{-x/\beta}$, $0 < x < \infty$, $\alpha > 0, \beta > 0$

(3)

(Voir p 156).

$$\ell(\theta) = \log \prod_1^n f(x_i;\theta) = C - \alpha n \log\theta + (\alpha-1)\log\prod x_i - \frac{\sum x_i}{\theta}$$

$$\ell'(\theta) = -\frac{\alpha n}{\theta} + \frac{\sum x_i}{\theta^2} = 0$$

$$\Longrightarrow \quad \hat{\theta} = \left(\sum x_i / n\alpha\right)$$

2. 6.1.2 a) $f(x;\theta) = \theta x^{\theta-1}$ $\quad 0 < x < 1 \quad 0 < \theta < \infty$

(4)

$$\ell(\theta) = n\log\theta + (\theta-1)\log\prod x_i$$

$$\ell'(\theta) = \frac{n}{\theta} + \log\prod x_i = 0$$

$$\Longrightarrow \quad \hat{\theta} = \frac{n}{\log(\prod x_i)}$$

b) $f(x;\theta) = e^{-(x-\theta)}$ $\quad \theta \le x < \infty \quad -\infty < \theta < \infty$

$$L(\theta) = \begin{cases} e^{-\sum(x_i-\theta)}, & \theta \le \min x_i < \infty \\ 0 & \text{ailleurs} \end{cases}$$

$$\Longrightarrow \quad \hat{\theta} = \min x_i$$

3. 6.1.11 $f(x;\theta) = \dfrac{\theta^x e^{-\theta}}{x!}$, $x = 0, 1, \dots$, $0 < \theta \leq 2$

(3)

$$L(\theta) = \frac{\theta^{\sum x_i} e^{-n\theta}}{\prod x_i!} \qquad 0 < \theta \leq 2$$

$$\ell(\theta) = (\sum x_i) \log \theta - n\theta + C$$

$$\ell'(\theta) = \frac{\sum x_i}{\theta} - n = 0 \Rightarrow \hat{\theta} = \bar{x}$$

Mais aussi $0 < \theta \leq 2 \Rightarrow$

$$\hat{\theta} = \min(\bar{x}, 2).$$

4. 6.1.8 On calcule $\bar{x}$ à partir de la table

(2)

$$\bar{x} = \frac{7(0) + 14(1) + 12(2) + 13(3) + 6(4) + 3(5)}{7 + 14 + 12 + 13 + 6 + 3}$$

$$= 2.109 \qquad \Rightarrow \qquad \hat{\theta} = 2.109$$

$$P(X = 2) = \frac{\theta^2 e^{-\theta}}{2!}$$

$$\hat{P}(X = 2) = \frac{(2.109)^2 e^{-2.109}}{2!} = 0.26990$$